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Four-dimensional gonihedric gauge spin system

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Abstract

We perform Monte Carlo simulations of a four-dimensional gauge invariant spin system which describes random surfaces with gonihedric action. We develop the analogy between the flat-crumpled phase transition of the lattice surface model and the liquid-gas phase transition of non-ideal gases, and identify the self-intersection coupling constant k of the surface model with the pressure P . As k increases the system moves to a critical point in complete analogy with the situation for non-ideal gases, where the liquid and the gas phases approach each other with increasing P . We measure vacuum expectation values of various operators and the corresponding critical indices.

1 Introduction

In this article we consider a model of two-dimensional random surfaces embedded into an Euclidean lattice Z^4 , where a closed surface is associated with a collection of plaquettes. The surfaces may have self-intersections in the form of four and six plaquettes intersecting on a link. The edges of the surface with self-intersections comprise the *singular part* of the surface. The edges of the surface where only two plaquettes are intersecting comprise the *regular part* of the surface.

Various models of random surfaces built out of plaquettes have been considered in the literature [1]. In this article we consider the so-called gonihedric model which has been defined in refs. [2, 3] and we compare it with the model with area action [4, 5]. The gas of random surfaces defined in [4] corresponds to the partition function with Boltzmann weights proportional to the total number n of plaquettes, that is to Nambu-Goto area functional. The gonihedric model of random surfaces will correspond to a statistical system with weights proportional to the total number n_2 of non-flat edges of the surface, that is to a linear size of the surface [2]. The edges are non-flat when the dihedral angle between plaquettes is not equal to π . The weights associated with self-intersections are proportional to kn_4 and kn_6 where n_4 and n_6 are the number of edges with four and six intersecting plaquettes, and k is the self-intersection coupling constant [2, 3].

To study the statistical and scaling properties of the system one can directly simulate surfaces by gluing together plaquettes following the rules described above, that is on every link only an even number of plaquettes should intersect and the weights are proportional to the area n or gonihedrality $n_2 + 4kn_4 + 12kn_6$. But it is much easier to use the duality between random surfaces and spin systems on Euclidean lattice. This duality allows us to study equivalent spin systems with specially adjusted interaction between spins [3].

In four dimensions a spin Hamiltonian which is equivalent to an area action, is well known. It represents a gauge invariant spin system with one-plaquette interaction terms [4]

$$H_{area}^{4d} = -\frac{1}{g^2} \sum_{\{plaquettes\}} (\sigma\sigma\sigma\sigma), \quad (1)$$

where spins are located on the links and the summation is over all plaquettes on the lattice. Likewise there exists a gauge invariant Hamiltonian which is equivalent to the gonihedric action. It has a more complicated interaction between spins [3]:

$$\begin{aligned} H_{gonihedric}^{4d} = & -\frac{5\kappa - 1}{g^2} \sum_{\{plaquettes\}} (\sigma\sigma\sigma\sigma) + \frac{\kappa}{4g^2} \sum_{\{right\ angle\ plaquettes\}} (\sigma\sigma\sigma\sigma_\alpha)^{rt} (\sigma_\alpha\sigma\sigma\sigma) \\ & - \frac{1 - \kappa}{8g^2} \sum_{\{triples\ of\ right\ angle\ plaquettes\}} (\sigma\sigma\sigma\sigma_\alpha)^{rt} (\sigma_\alpha\sigma\sigma\sigma_\beta)^{rt} (\sigma_\beta\sigma\sigma\sigma), \end{aligned} \quad (2)$$

where the spins interact within a three-dimensional cube of the four-dimensional lattice. The coupling constant k monitors the interaction at the singular parts of the surface where self-intersections take place. If k is equal to zero then there is no repulsion associated with self-intersections, and if k is large the surfaces are self-avoiding [3].

The partition functions for both systems have the form

$$Z(\beta) = \sum_{\{\sigma\}} e^{-\beta \ g^2 H/4},$$

where the summation is over all spin configurations. This partition function can be represented in a dual form as a sum over two-dimensional surfaces of the type described above, embedded into a four-dimensional lattice [2, 3]:

$$Z(\beta) = \sum_{\{\text{surfaces } M\}} e^{-\beta \epsilon(M)} \quad (3)$$

where $\epsilon(M)$ is the energy of the surface M constructed from plaquettes such that each edge of the surface is contained in either two, four or six plaquettes. Those edges of the surface where four and six plaquettes intersect comprise the curves of self-intersections.

If a two-dimensional surface M has n plaquettes, n_2 edges with two-plaquette intersection, n_4 edges with four-plaquette intersection (more precisely there are two different geometries of four-plaquette intersections, the corresponding numbers are equal to \bar{n}_4 and $\bar{\bar{n}}_4$ [3]) and n_6 edges with six-plaquette intersection, then the total energy $\epsilon(M)$ of the surface M with Hamiltonian (1) is equal to n :

$$\epsilon_{\text{area}}(M) = n(M).$$

For the gonihedric system with Hamiltonian (2) the energy is related to the number of non-flat edges of M by:

$$\epsilon_{\text{gonihedric}}(M) = n_2(M) + 4k\bar{n}_4(M) + (6k - 1)\bar{\bar{n}}_4(M) + 12kn_6(M). \quad (4)$$

If k is small self-intersections are permitted, but when k increases there is strong repulsion along the curves of self-intersections, and the surfaces tend to be self-avoiding [3].

The total energy is defined as

$$E = -\partial \ln Z / \partial \beta = \langle n_2 + 4k\bar{n}_4 + (6k - 1)\bar{\bar{n}}_4 + 12kn_6 \rangle_c$$

and specific heat as

$$C = \beta^2 \frac{\partial^2 \ln Z}{\partial \beta^2} = -\beta^2 \frac{\partial E}{\partial \beta}. \quad (5)$$

The derivative of the partition function (3) with respect to k gives an *average length* μ of the self-intersection curves

$$\mu = -\frac{1}{\beta} \frac{\partial \ln Z}{\partial k} = \langle 4\bar{n}_4 + 6\bar{\bar{n}}_4 + 12n_6 \rangle_c \quad (6)$$

which we suggest to consider as a *disorder parameter*, because it is zero in the low-temperature phase and is nonzero at high-temperature phase [8]. The second derivative with respect to k defines the *intersection susceptibility*

$$\chi = \frac{1}{\beta^2} \frac{\partial^2 \ln Z}{\partial k^2} = -\frac{1}{\beta} \frac{\partial \mu}{\partial k} = \langle (4\bar{n}_4 + 6\bar{\bar{n}}_4 + 12n_6)^2 \rangle_c - \langle 4\bar{n}_4 + 6\bar{\bar{n}}_4 + 12n_6 \rangle_c^2. \quad (7)$$

The average area S and the area susceptibility χ^S are defined analogously:

$$S = \langle n \rangle_c, \quad \chi^S = \langle n^2 \rangle_c - \langle n \rangle_c^2.$$

All these observables can be expressed in terms of spin variables and then used in Monte Carlo simulations [3, 8].

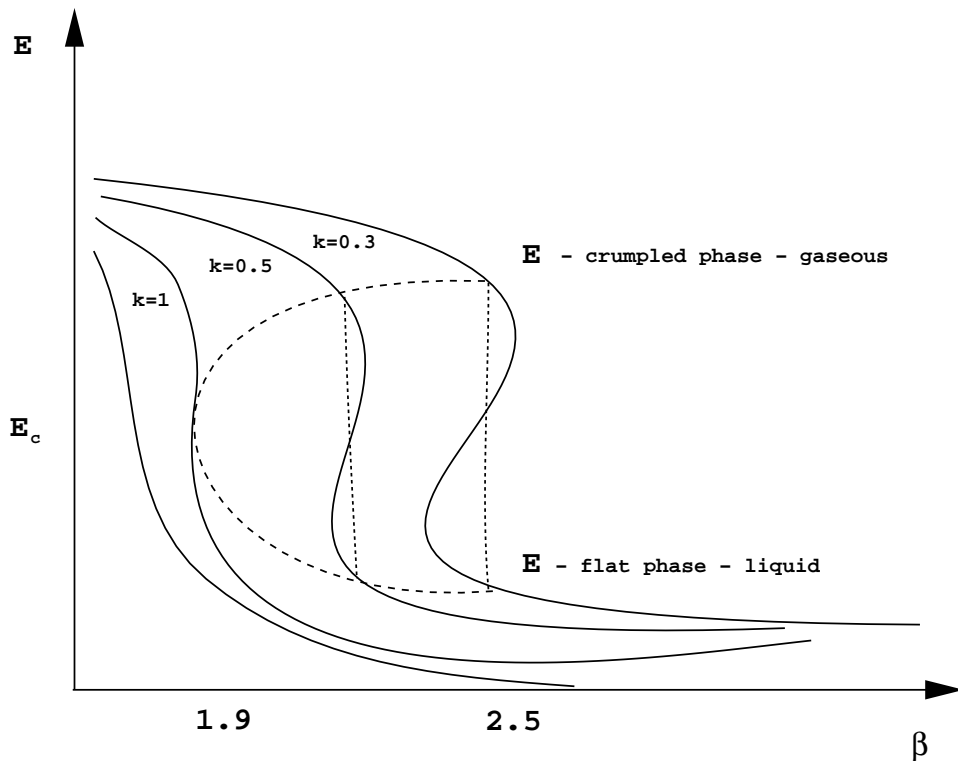


Figure 1: Schematic exposition of the equation of state for the 4D-system (2), which is similar to Van der Waals-like curves and coexistence curve for the real gases. The system was heated and then cooled in small β steps [8].

In [8] it was demonstrated that the system, with a sufficiently large value for the self-intersection coupling constant k ($k_c \approx 1$), undergoes a second-order phase transition at some critical temperature β_c . This result suggests the existence of continuum field theory in four dimensions and our aim is to study further the phase structure of the system, the scaling properties of different operators, and to compute the corresponding critical indices. For this purpose it is helpful to develop the analogy between flat-crumpled phase transition of the gonihedric surface model [3, 8] and liquid-gas phase transition of non-ideal gases, and to identify the intersection coupling constant k with the pressure P , as well as the intersection susceptibility (7) with compressibility K_T [9].

2 Analogy with non-ideal gases

Before presenting new results for gonihedric system let us describe the known behaviour of spin systems with an area-type action [6]. As mentioned in the introduction, the random surfaces with an area action can be represented by the gauge invariant Hamiltonian (1) [4]. In four dimensions this gauge invariant spin system is self-dual and the critical temperature is equal to $\beta_c = \frac{1}{2} \ln(1 + \sqrt{2})$ [4]. Monte Carlo simulations strongly suggest that the phase transition in 4d Z_2 gauge invariant spin system is of first order [6, 7, 10]. A clear signal of the first order nature is obtained by measuring the one-plaquette average in a thermal cycle.

A thermal cycle of the statistical system provides a general overview of its phase

structure and is very helpful in defining the regions of the phase transitions [6], and pronounced hysteresis is strongly indicative of a first-order phase transition [6]. A typical hysteresis curve will look like the curve corresponding to $k = 0.3$ on Fig. 1, and is due to the meta-stability of the ordered phase at high temperature and of the disordered phase at low temperature.

The phase structure of the self-avoiding random surface model in four dimensions, with the gonihedric action (2), has recently been analysed in [8]. The Monte Carlo simulations of the corresponding gauge invariant spin system demonstrate that the critical behaviour of the system essentially depends on the self-intersection coupling constant k . For small values of k a thermal cycle shows a clear hysteresis loop, thus favouring a first order phase transition (see the Van der Waals-like curves $k=0.3$ and $k=0.5$ on Figure 1). Two distinct stable phases appear at the critical temperature with large differences between their energy densities:

$$\delta E = E_{crumpled} - E_{flat}, \quad (8)$$

(where the names *crumpled* and *flat* refer to the typical embedding of the surfaces on the hyper-cubic lattice), see Figure 1 and Figures 1-9 in Ref.[8]. This first order phase transition at small k is very similar to the first order phase transition in the model with the one-plaquette area action (1).

We have to define also a gap in the intersecting energy μ

$$\delta\mu = \mu_{crumpled} - \mu_{flat}, \quad (9)$$

and a gaps in the simple bendings n_2 and the average area S as

$$\delta n_2 = n_{2\ crumpled} - n_{2\ flat}, \quad \delta S = S_{crumpled} - S_{flat} \quad (10)$$

As the self-intersection coupling constant k increases, the energy difference δE – the gap between the densities $E_{crumpled}$ and E_{flat} of coexisting phases – tends continuously to zero (see Figure 2 and Table 1). The same is true for the gaps in the cases of intersecting energy, of simple bendings and of average area (see Figure 2 and Table 1). These gaps tend to zero also when the volume increases for the fixed value of the coupling constant $k_c \approx 1$, as it can be seen from Table 1.

TABLE 1

Volume dependence of the total energy and intersection energy:

k	Volume	β_{crit}	C_{max}/V	δE	χ_{max}/V	$\delta\mu$
0.3	6^4	2.64-2.66	0.101(3)	0.64(2)	0.0133(6)	0.23(1)
0.3	10^4	2.50-2.55	0.108(5)	0.66(3)	0.015(1)	0.25(2)
0.3	12^4	2.50-2.54	0.10(1)	0.64(6)	0.014(2)	0.24(3)
0.3	20^4	2.50-2.54	0.1050(8)	0.648(5)	0.01471(8)	0.243(1)
1.0	6^4	1.840-1.849	0.018(5)	0.26(6)	0.00021(7)	0.003(1)
1.0	8^4	1.904-1.906	0.015(2)	0.24(3)	0.00010(2)	0.0019(5)
1.0	10^4	1.919-1.920	0.011(2)	0.20(3)	0.00007(1)	0.0017(3)
1.0	12^4	1.920-1.921	0.009(2)	0.18(4)	0.00007(1)	0.0016(2)
1.0	16^4	1.915-1.920	0.009(1)	0.19(2)	0.00006(1)	0.0016(2)
1.0	20^4	1.925-1.926	0.008(2)	0.17(4)	0.00006(1)	0.0015(3)

Volume dependence of the simple bendings and the plaquettes:

k	Volume	β_{crit}	$\chi_{max}^{n_2}/V$	δn_2	χ_{max}^S/V	δS
0.3	6^4	2.64-2.66	0.041(2)	0.41(1)	0.08(3)	0.6(2)
0.3	10^4	2.50-2.55	0.042(2)	0.41(1)	0.145(4)	0.76(2)
0.3	12^4	2.50-2.54	0.041(3)	0.41(2)	0.146(9)	0.76(5)
0.3	20^4	2.50-2.54	0.0405(4)	0.405(4)	0.141(1)	0.754(5)
1.0	6^4	1.840-1.849	0.018(4)	0.26(6)	0.005(2)	0.15(4)
1.0	8^4	1.904-1.906	0.013(2)	0.22(3)	0.007(2)	0.16(3)
1.0	10^4	1.919-1.920	0.009(1)	0.19(3)	0.008(1)	0.15(2)
1.0	12^4	1.920-1.921	0.0075(7)	0.17(2)	0.0085(8)	0.16(2)
1.0	16^4	1.915-1.920	0.008(1)	0.18(2)	0.010(2)	0.20(3)
1.0	20^4	1.925-1.926	0.007(1)	0.16(3)	0.008(2)	0.17(4)

The first order phase transition becomes weaker and then disappears. The limiting density E_c and the corresponding temperature β_c and the coupling constant k_c define the critical point of a second-order phase transition. Thus the intersection coupling constant k plays an important role and by monitoring k one can drive the system to a critical point where there might be a second order transition.

This picture of the phase transition of the four-dimensional system has a great similarity with the liquid-gas critical point in three dimensions, and the coexistence curves on Fig. 1 and 2 is the analogue of the coexistence curve of the liquid and the gas phases. We will associate the flat phase of the low temperature gauge system with the liquid phase and the crumpled high temperature phase with the gaseous phase. In accordance with the proposed analogy, the self-intersection coupling constant k should be associated with the pressure P.

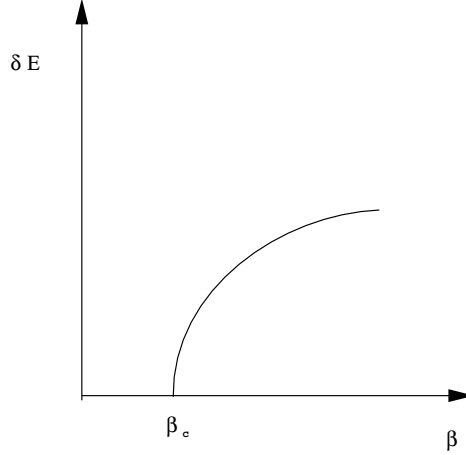


Figure 2: Coexistence curve: plot of the gap $\delta E = E_{crumpled} - E_{flat}$ vs $(\beta - \beta_c)$. The gap vanishes as $(\beta - \beta_c)^{1-\alpha}$

Indeed, the non-ideal gas condenses to a liquid state by cooling at fixed pressure. As pressure increases condensation takes place at higher temperatures and two phases merge at the critical point. The same phenomenon takes place in the gonihedric system in four-dimensions. By heating the low temperature flat-liquid phase at fixed k we “evaporate” clouds of crumpled surfaces (see Figure 1) at some temperature β_k . As the self-intersection coupling constant k increases evaporation of the clouds takes place at higher and higher temperatures (see Figure 1 and Table 1) and the energy gap between two phases decreases and tends to zero (see Figure 2). Thus the self-intersection coupling constant k allows us to move the system from one Van der Waals-like curve to another and finally to the critical point. In the case of the area-action the system does not have such an additional coupling constant and is described by single Van der Waals-like curve [6].

The fact that the clouds of evaporated surfaces are indeed crumpled can be seen from the self-intersection energy density $\epsilon_{intersection}$. This part of the total energy $\epsilon_{gonihedric}$ is equal to zero at the low temperature flat phase, and jumps to a nonzero value at the evaporation point β_k [8]. As k increases the evaporation takes place at higher temperatures $\beta_{k_1} < \beta_{k_2}$ ($k_2 < k_1$) and two phases merge at the critical point $(\beta_c, k_c) \approx (1.925, 1)$. The curve of the first-phase transitions in the (β, k) coupling constant plane ends at the critical point (β_c, k_c) as illustrated in Fig. 3.

3 Monte Carlo simulation of gonihedric system

Before describing the results, let us briefly explain some points having to do with the simulation. One of the basic issues is the order of the phase transition, which has to be determined by the study of the volume dependence of several quantities, namely the peaks of the variances $\Delta Q^2 \equiv \langle Q^2 \rangle - \langle Q \rangle^2$ (specific heat C , intersection susceptibility χ , area susceptibility χ_S) of the measured quantities Q (total energy E , average length μ , area S). We recall that for a first order transition the peak divided by the volume $\delta Q^2|_{max} \equiv \frac{\Delta Q^2|_{max}}{V}$ should be volume independent; on the other hand, for a continuous transition, the volume dependence will be weaker and should be characterized by critical exponents: $\alpha/d\nu$ for the specific heat C , $\gamma/d\nu$ for the intersection susceptibility and $\tilde{\gamma}/d\nu$

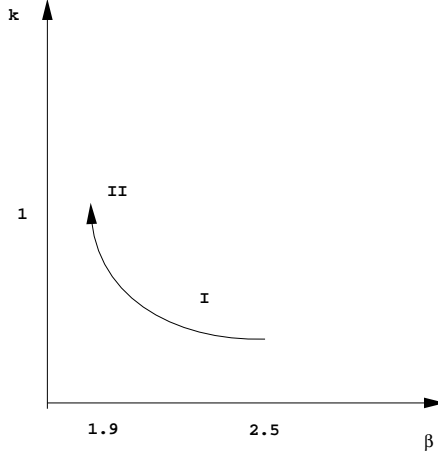


Figure 3: The curve of the first-order phase transitions in the (β, k) coupling constant plane is ended at the critical point (β_c, k_c) .

for the area susceptibility χ_S

$$\Delta Q^2|_{max} \approx V^{index}. \quad (11)$$

A quantity related to the variance is the so called gap. While simulating the system we found out that if we average over only one of the meta-stable states, the resulting values $\langle Q \rangle_{crumpled}$, $\langle Q \rangle_{flat}$ (for the quantity Q , say) were well separated; the intermediate values have rarely shown up. One may define the gap $g(Q)$ as the difference: $g(Q) \equiv \langle Q \rangle_{crumpled} - \langle Q \rangle_{flat}$. It is easy to show that, in the situation just described, $g(Q)$ is related to the peak of the variance through $\delta Q^2|_{max} \approx \frac{g^2(Q)}{4}$. Thus, if the phase transition is of first order, the lack of volume dependence for $\delta Q^2|_{max}$ will result to a volume independence of $g(Q)$ as well. Thus, the volume independence of the gap may serve as an additional indicator of the order of the phase transition. We note that for a continuous phase transition the gap will decrease for increasing volume.

We have performed Monte Carlo simulations at several values of k and β close to the phase transition lines that have been determined in [8]. We choose to show in the sequel the results for $k = 0.3$ and $k = 1.0$ for the lattice sizes 6^4 , 8^4 , 10^4 , 12^4 , 16^4 , 20^4 . The observables we have measured are the following:

- The total energy $H_{gonihedric}^{4d}$, as defined in equation (2) and (4).
- The part of the energy related to the simple bendings of plaquettes (n_2).
- The intersection energy, which is defined as the difference of the two previous quantities (6) .
- The plaquette (n).

Table 1 displays the results of the measurements. For $k=0.3$ or $k=1.0$ the critical values for β are reported for the various lattice volumes together with the peaks of the variances and the energy gaps (uncertainty in the last digit is put in parentheses). In the upper half of the table one finds the results for the total energy and for the interaction energy; in the lower half one finds the the results for the simple bendings and for the plaquette. Let us first observe that for $k=0.3$ the gap and the quantities $\delta Q^2|_{max}$ are

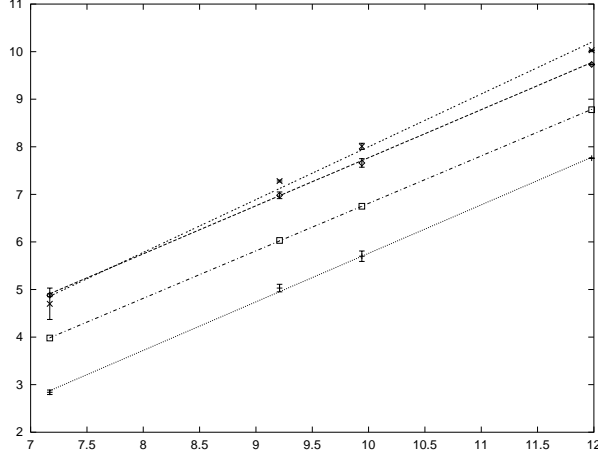


Figure 4: The critical indices at $k=0.3$. All quantities scales with the lattice volume as a first power. Intersections are represented by crosses, bendings by boxes, plaquettes by diamonds and energy by x's.

volume independent within errors, indicating that for this k the phase transition is of first order; one may also check that the relation $\delta Q^2|_{max} \approx \frac{g^2(Q)}{4}$ is approximately satisfied. For $k=0.3$ the total energy and the simple bendings have a sizeable difference which shows up as intersection energy. We should remark that the volume independence of the gap and the peak of the variance also holds for $k=1.0$ in the special case of the plaquette.

Having found the relevant $\Delta Q^2|_{max}$ we can extract the critical exponents using (11).

TABLE 2 Coefficients a and b of the best fit $a+b x$

k		a	δa	b	δb
0.3	Energy	-2.34	0.07	1.008	0.006
0.3	Intersections	-4.4	0.1	1.02	0.01
0.3	Bendings	-3.16	0.08	0.997	0.007
0.3	Plaquette	-1.88	0.09	0.993	0.007
1.0	Energy	-2.9	0.5	0.83	0.05
1.0	Intersections	-7.9	0.6	0.83	0.06
1.0	Bendings	-2.9	0.4	0.82	0.04
1.0	Plaquette	-5.8	0.5	1.11	0.05

The results are depicted in Fig. 4. The values of $\log(\Delta Q^2|_{max})$ are plotted versus the logarithm of the volume for the four measured quantities. Fig. 4 also contains the best straight line fits. The best parameters (with their errors) are collected in Table 2. It is striking that for this small k the slope is equal to one within the errors. This means that the variance is proportional to the volume, that is the ratio $\delta Q^2|_{max} \equiv \frac{\Delta Q^2|_{max}}{V}$ is a constant, which is once more the sign of a first order phase transition.

For $k=1.0$ both the gap and $\delta Q^2|_{max}$ decrease with increasing volume, thus indicating that the transition has become second order. One should also notice that for $k=0.3$ the total energy and the bendings have a sizeable difference, which shows up as intersection energy; on the contrary, for $k=1.0$ the energy is almost entirely built up from simple

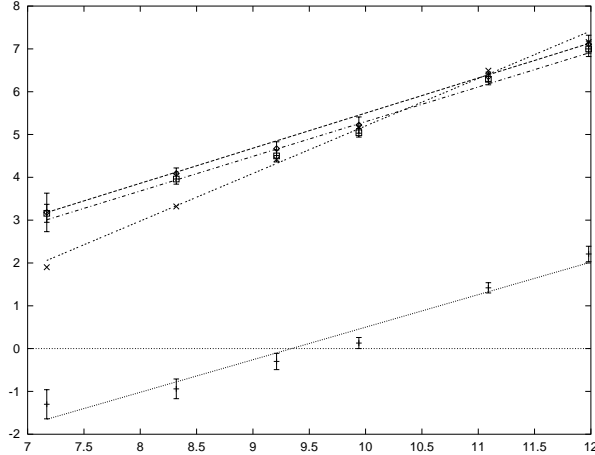


Figure 5: The critical indices at $k=1$.

bendings and plaquettes, which is reflected to the very small value of the intersection energy. One should also notice the quite large critical exponent for the plaquette.

To substantiate these claims, we plot in Figure 5 $\text{Log}(\Delta Q^2|_{max})$ versus the logarithm of the volume and determine the relevant critical exponent. We find that the best slopes are around 0.83 for all quantities with the exception of the plaquette. The small value of the exponent, as compared against 1.0 above, also lends support to the view that the transition is continuous.

Thus we conclude that finite size scaling favours a second order phase transition at (β_c, k_c) with seemingly nontrivial exponents (but see next section), and it is apparent that the transition has great qualitative similarity with the liquid-gas second order transition at the coexistence point (β_c, P_c) .

4 Discussion

We have analyzed numerically the four-dimensional goniuhedric gauge spin system. In its random surface representation it exhibits a flat-crumpled transition which is in close analogy with the liquid-gas transition of non-ideal gases, and with the intersection coupling constant k of the spin system playing the role of the pressure P . We found evidence that in the neighborhood of $k = 1.0$ the first order transition between the flat and crumpled phase changes to a second order transition. Again, this is in accordance with the liquid-gas picture. We found $\alpha/d\nu = 0.83 \pm 0.06$ (except for the plaquette energy which has $\alpha/d\nu \approx 1.11$). It could indicate a non-trivial scaling and thus a new four-dimensional class of critical systems. However, before jumping to these conclusions one should keep in mind that it is notoriously difficult to measure the exponents at a point where the first order transition stops and changes to a higher order transition. The true exponents tend to be masked by so-called pseudo-critical exponents of the first order transition: $\nu = 1/d$, where d is the dimension of space [11]. In our case it would mean that a pseudo-critical ν should be 0.25 if we did measure the first order pseudo-critical exponent. From the measured values of $\alpha/d\nu$ and assuming hyper-scaling ($2 - \alpha = \nu d$) we get $\nu = 0.27 \pm 0.01$ for ΔQ^2 coming from energy, intersections and bendings. These values seem uncomfortable close to $\nu = 0.25$ and although the measured value of $\alpha/d\nu$ is more that two standard

deviations away from the first-order value $\alpha/d\nu = 1$, systematic errors might be larger than anticipated. Thus we cannot rule out that we see a very slow shift away from this value and towards the true value for the second order transition, even if we have carefully checked for finite volume dependence. If this is the case the true value of the critical exponent is of course unknown. Further studies seem necessary in order to settle this interesting question in an unambiguous way.

5 Acknowledgement

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